On Analyzing Estimation Errors due to Constrained Connections in Online Review Systems

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Background and the Problem

- Online Review Systems
  - Reviewers: spammer/non-spammer
  - Items: good/bad
- The *labels* of reviewer/item can be co-inferred using rating data.
- However, most of the works ignore the effects of online review system *topologies*,
  - i.e., the bipartite graph representing which reviewers can review which items.
- **Constrained Connections**: A reviewer usually only reviews a few items due to limited interest scope, attention capacity etc.

In this work

How constrained connections can affect the estimation performance in online review systems?

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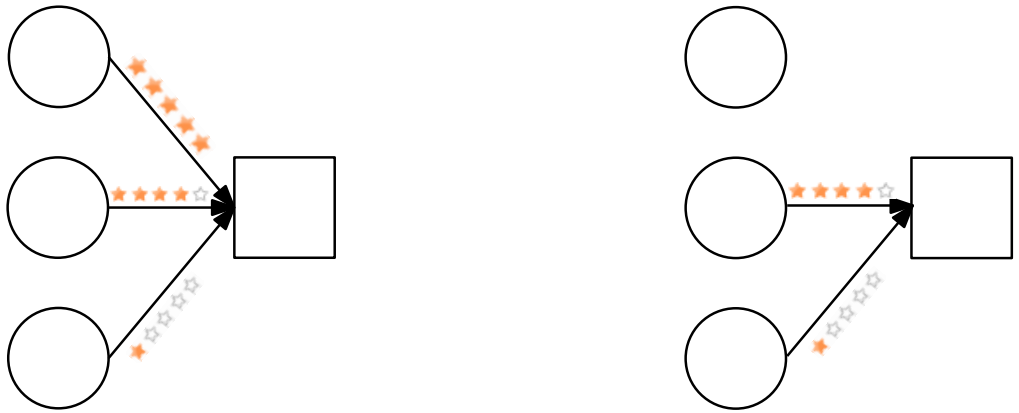
Why Constrained Connections can Affect Estimation?

An Intuitive Example



- In Figure (a), we can infer with high confidence that the item is probably good, and the bottom reviewer is likely to be a spammer.
- In Figure (b), due to constrained connections, we cannot give high confidence conclusions.

Data Model

- A set of reviewers V and a set of items I.
- Each item $i \in I$ has an unknown binary label $z_i \in \{\pm 1\}$.
- A review r_{ui} represents reviewer u's attitude to item i, e.g., $r_{ui} = +1$ if u thinks i to be good.
- Each reviewer u makes correct reviews with probability θ_u, and $P(\theta_u) \propto \theta_u^{\alpha-1}(1-\theta_u)^{\beta-1}$, where $\alpha > \beta$.
- Reviewer u can review item i if there is an edge $(u, i) \in E$ in bipartite graph $G(V, I, E)$.
- Given a collection of n reviews $R_n = \{r_1, \cdots, r_n\}$, our goal is to study how G affects the estimation of $\theta = \{\theta_u\}_{u \in V}$ and $z = \{z_i\}_{i \in I}$.

Maximum A Posteriori Estimator (MAPE)

- We treat θ as parameters and z as hidden variables, and use EM algorithm to estimate them:

$$\textbf{Objective: } \max_{\theta} \log P(\theta|R_n) = \max_{\theta} \log \sum_{z} P(\theta, z|R_n)$$

$$\textbf{E-Step: } \mu_i(z_i) \equiv P(z_i|R_{\cdot i}, \theta) \propto P(z_i) \prod_{u \in V_i} \theta_u^{n_{ui}^{z_i}} (1-\theta_{ui})^{n_{ui}^{-z_i}},$$

$$\textbf{M-Step: } \theta_u^{(t+1)} = \frac{\sum_{i \in I_u} \sum_{x \in \{\pm 1\}} n_{ui}^{x} \mu_i(x) + \alpha - 1}{|R_{u\cdot}| + \alpha + \beta - 2}.$$

- The EM algorithm implicitly defines an estimator of θ, i.e., $\hat{\theta}_{\text{MAP}} = \text{EM}(R_n)$.

Estimation Errors due to G

- To understand how G affects $\hat{\theta}_{\text{MAP}}$, we can calculate MSE of $\hat{\theta}_{\text{MAP}}$ over different graphs. ($\text{MSE}(\hat{\theta}_u) = \mathbb{E}\left[\hat{\theta}_u - \theta_u\right]^2$)
- MSE of $\hat{\theta}_{\text{MAP}}$ is lower bounded by the Bayes Cramer-Rao bound requiring that the estimator is weakly unbiased.
- The posterior distribution of θ can be approximated by normal distribution, i.e.,

$$P(\theta|R_n) \to \mathcal{N}(\hat{\theta}_{\text{MAP}}, \mathcal{I}(\hat{\theta}_{\text{MAP}})^{-1}) \quad \text{as } n \to \infty,$$

where $\mathcal{I}(\hat{\theta}_{\text{MAP}})$ is the *observed Fisher information matrix*, and each element is

$$\left[\mathcal{I}(\hat{\theta}_{\text{MAP}})\right]_{uv} = -\frac{\partial^2 \log P(\theta|R_n)}{\partial \theta_u \partial \theta_v}\Big|_{\theta=\hat{\theta}_{\text{MAP}}}.$$

Obtaining the Observed Fisher Information Matrix

Theorem

The observed Fisher information matrix is a diagonal matrix, with each diagonal element given by

$$\mathcal{I}_{uu} = \frac{\alpha - 1}{\hat{\theta}_u^2} + \frac{\beta - 1}{(1 - \hat{\theta}_u)^2}.$$

- Given hyper-parameters α and β, θ_u is most uncertain at $\frac{1}{1+\sqrt[4]{(\beta-1)/(\alpha-1)}}$, and most certain at 0 or 1.

Experiments and Observations I

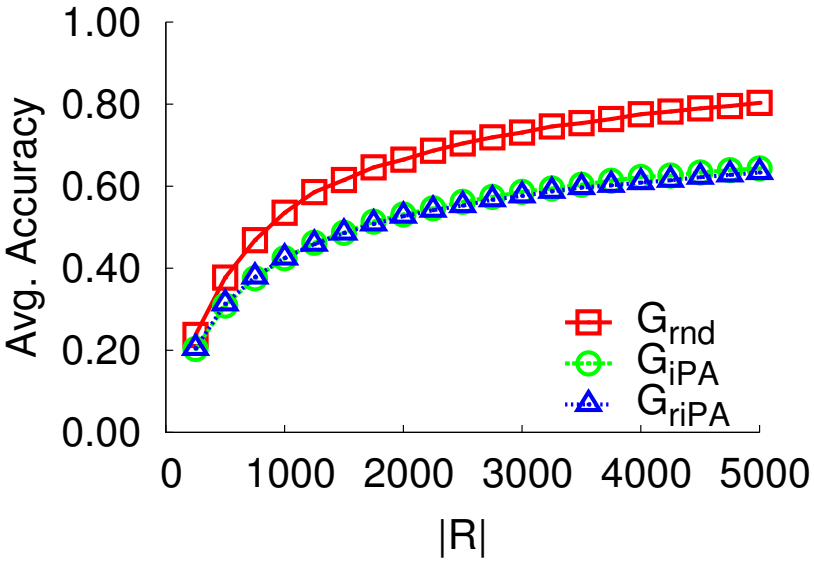
Bipartite Graph Models

- Random Model G_{rnd}:
  - Each edge (u, i) is formed by randomly connecting a reviewer u and item i.
- Item Preferential Attachment Model G_{iPA}:
  - For edge (u, i), u is chosen randomly, and i is chosen with probability proportion to i's degree.
  - This mimics real-world situation that popular items are more likely to be reviewed.
- Reviewer and Item Preferential Attachment Model G_{riPA}:
  - For edge (u, i), u and i are chosen with probability proportion to their degrees in the graph.

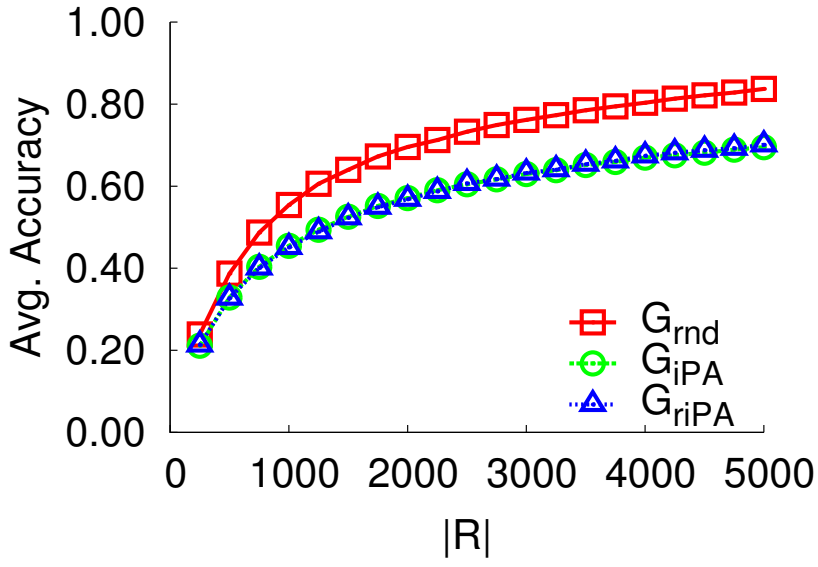
Remarks:

- From G_{rnd} to G_{iPA} and G_{riPA}, we put more constraints on the topology.
- How do constraints affect estimation errors?

Experiments and Observations II



(a) $|E| = 2000$

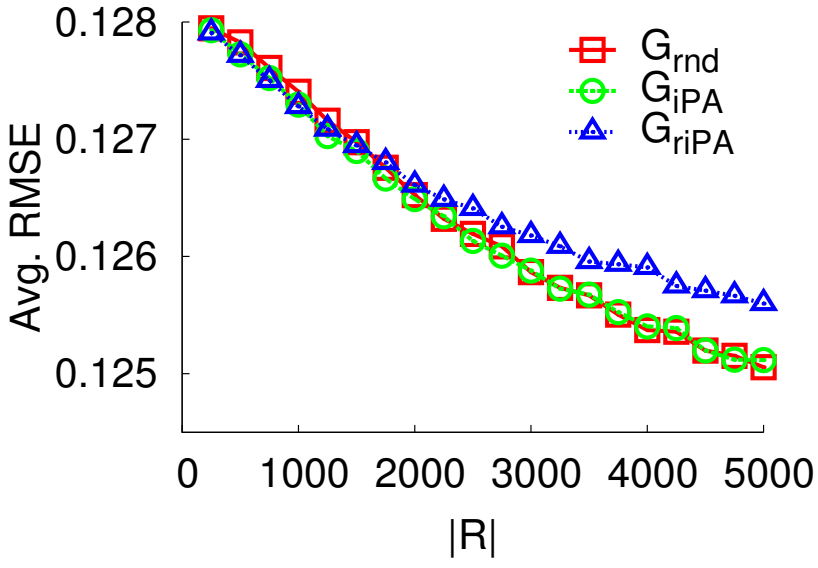


(b) $|E| = 3000$

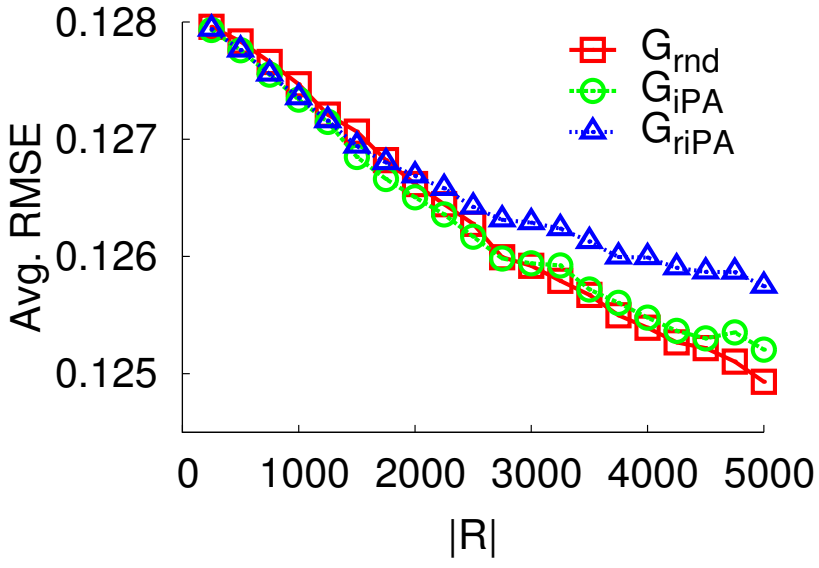
Estimation Accuracy Curve:

- Accuracy is the fraction of items can be correctly inferred.
- MAPE is asymptotically unbiased.
- Accuracy on G_{rnd} is larger than the others: **more constraints make estimation accuracy poor**.

Experiments and Observations III



(c) $|E| = 2000$



(d) $|E| = 3000$

RMSE (RMSE $= \sqrt{\text{MSE}}$) Lower Bound:

- The RMSE decreases approximately with rate $\frac{1}{n}$.
- When n is large, RMSE on G_{riPA} is largest, then comes G_{iPA} and RMSE on G_{rnd} is the lowest: **more constraints implies harder to be estimated.**

Conclusion and Future Work

- More constraints on the topology usually cause worse estimation performance.
- Suppose the constrains are **fixed**, is there an optimal topology that has the lowest RMSE? How can we find this topology?

Thank you!

Feel free to contact me if you have any questions:
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